Quantum Model of a Rat in the Maze

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Intro: why rat, why quantum?

Quantum mechanics (QM) provides a formal framework for modelling uncertainty and dynamic evolution in physical systems. While its mathematical structure is well established, its application beyond microscopic phenomena remains an area of active discussion. This work attempts to illustrate the axioms of QM in an intuitive and accessible way on a textbook example. The poster presents an approach alternative to [3], starting from the axiomatics of QM as stated in [4]. Main focus is on interpretational aspects (how to understand the mathematics). Although observation process in QM works both for conscious and unconscious observer in the same way, we expect that future deeper analysis of the living observer's perspective can help to narrow the existing gap between decision making and QM. Extended paper is in preparation.

Setup: state space

A closed non-transparent box has several rooms (Fig.1). A rat is moving between the rooms in discrete steps (position of the rat is represented by room number). The rat is a normal rodent, she cannot physically be in several rooms at once. Only some rooms share common door. The rat can only move through one door at a time (i.e. it takes her several steps to reach distant rooms). The rat starts in Room 1 (initial condition) and she prefers to stay around Room 5, since it's the room where she is usually fed (she is naturally attracted to that room).

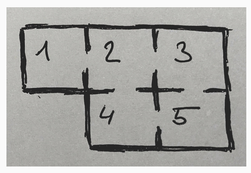


Figure 1: Scheme of the maze

- Since the box has 5 rooms, the rat's state can be represented as a vector in 5-dimensional Hilbert space $\mathcal{H} = \text{span}\left\{|\text{Room}_1\rangle, |\text{Room}_2\rangle, |\text{Room}_3\rangle, |\text{Room}_4\rangle, |\text{Room}_5\rangle\right\}$.
- The state of the rat can be expressed as a linear combination:

$$|\psi(t)\rangle = c_1 |\text{Room}_1\rangle + c_2 |\text{Room}_2\rangle + c_3 |\text{Room}_3\rangle + c_4 |\text{Room}_4\rangle + c_5 |\text{Room}_5\rangle,$$

  with coefficients $c_i \in \mathbf{C}$ satisfying the normalization $\sum_{j=1}^{5} |c_j|^2 = 1$.
  Numeric representation (basis in $\mathcal{H}$) can be chosen as unit vectors: $|\text{Room}_j\rangle \equiv |e_j\rangle$.
- Since the state is changing in time t, the coefficients are time dependent: $c_j(t)$.

Observation

- Performing a measurement (an observation) means observing position of the rat. Possible outcomes: {'1', '2', '3', '4', '5'}.
- Observable is Hermitian operator $\hat{\text{A}} : \mathcal{H} \to \mathcal{H}$ represented by matrix $\mathbb{A} \in \mathbf{R}^{5\times5}$ such that its spectrum consists of possible outcomes.
- **Probability of observing outcome:** since the rat's movement is stochastic, we can only think about the probability of finding rat in the j-th room if the system is in state $|\psi(t)\rangle$. According to Born's rule:

$$P\left(\text{rat is in the } j\text{-th room}\middle|\, |\psi(t)\rangle\right) = P\left(\alpha_j \middle|\, |\psi(t)\rangle\right) = \left\| \hat{\text{E}}(\alpha_j) |\psi(t)\rangle \right\|^2,$$

  where α_j is the j-th eigenvalue (representing the j-th outcome), and $\hat{\text{E}}(\alpha_j) : \mathcal{H} \to \mathcal{H}$ is a projector to direction $|e_j\rangle$: $\hat{\text{E}}(\alpha_j) = |e_j\rangle\langle e_j|$. It also gives a new meaning to c_j: $P\left(\text{rat is in the } j\text{-th room}\middle|\, |\psi(t)\rangle\right) = |c_j|^2$.
- **State update by an observation**: the observation process influences system's state via Lüders formula (a.k.a. Projection postulate of QM):

$$|\psi_{\alpha_j}(t)\rangle = \frac{\hat{\text{E}}(\alpha_j) |\psi(t)\rangle}{\left\| \hat{\text{E}}(\alpha_j) |\psi(t)\rangle \right\|}. \tag{1}$$

  This shows discontinuous change of the system's state.

Time Evolution

Between two observations we don't receive any information about rat movement. The rat's state changes according to the Schrödinger equation or equivalently by unitary operator $\hat{\text{U}} : \mathcal{H} \to \mathcal{H}$ in the following way:

$$|\psi(t)\rangle = \hat{\text{U}}(t) |\psi(0)\rangle, \quad |\psi(0)\rangle = |\psi_0\rangle, \tag{2}$$

where $|\psi_0\rangle \in \mathcal{H}$ is initial condition and $\hat{\text{U}}(t) = \exp\left(-it\frac{1}{\hbar}\hat{\text{H}}\right)$, with $\hat{\text{H}}$ denoting Hamiltonian of the system (represented by a Hermitian matrix).

🤔 How to choose $\hat{\text{H}}$ for such system?

In our experiment, we perform obsrevations in regular intervals (every N time steps). After each observation, state is updated (1) and system evolves again (2).

Choice of Hamiltonian

The following choice of $\hat{\text{H}}$ has been made by intuition and needs to be verified.

- The **diagonal elements** represent the energy associated with a state (room). The rat preferes the states with the smaller "energy". It can be shown that only differences between energy levels matter.
- The **off-diagonal elements** represents coupling strength of the states, in case of the rat – how likely is rat going through the door (transition effort between two rooms). Mainly this should reflect which rooms do not have a direct connection.

One of possible choices (where $a \in \mathbf{R}$ is expresses the connection of rooms):

$$\hat{\text{H}} = \begin{pmatrix} 3 & a & 0 & 0 & 0 \\ a & 2 & a & a & 0 \\ 0 & a & 1 & 0 & a \\ 0 & a & 0 & 1 & a \\ 0 & 0 & a & a & 0 \end{pmatrix}. \tag{3}$$

🤔 What the values should be? Does proper rule exist?
🤔 Can elements in (3) depend on external parameters (history, mood, ...)?
🤔 Can the elements be understood as rewards of the rat?

Simulation

Fig. 2 shows preliminary results of a simulation (in Python).The results are enormously affected by elements in $\hat{\text{H}}$. Several experiments were done, both with small and large off-diag.elements; negative values were also tested. Frequency of observing (every 50 steps observation is performed) also influences the results.

🤔 How to choose opt. frequency of observations? (too high ⇒ rat will be stuck in one room, a.k.a. Quantum Zeno Effect)
🤔 Min. disturbance via observation & max. information obtained - ?
🤔 Should be analysed frequencies or transitions of observations?

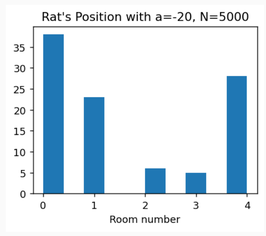


Figure 2: Observed positions.

Interpretation

1. $|\psi(t)\rangle$ **expresses real nature of the system:** rat is some "gas" occupying partially every room in maze at once. When observed, rat "condenses" in a single room (state collapse).
   - 👍 consistent with Copenhagen interpretation
   - 👎 unrealistic; free will of the rat is hardly imaginable
2. **no reality until observation:** observed objects exist only at the moment of observation (when closed, the box should be considered empty).
   - 👍 no objective reality, any reality is acceptable; consistent with [2]
   - 👎 discards any hope of providing an explanation of how the nature works
3. **state vector is the observer's mental state regarding the system**: $|\psi(t)\rangle$ describes the state of the system relatively to the observer's mind, i.e. $|\psi(t)\rangle$ does not express real physical properties, but the observer's opinion about them, possibly the observer's expectations/ideas regarding the system. When observation is performed, it influences observer's information about the system (consistent with [1]). We claim that:
   - Influence of observation onto system is negligible in macro-scaled systems.
   - There can be a direct causal link between the observation and discontinuous state change (collapse), but it should be understood as observer's information gain, not a physical change in the system [3].
   - 👍 coincides with common human experience: gaining information
   - 👎 unusual from physical perspective

Summary

Even such a trivial example opens a variety of questions concerning interpretation and understanding the model behind. This model *does not offer new contributions* to QM theory but serves as a **conceptual bridge**, demonstrating its fundamental principles in a setting that includes a living organism. The approach may be useful in educational contexts or as a stepping stone for exploring quantum-inspired models in decision-making and behaviour modelling of living matter.

Future of the rat

- Two observables: the 2nd observable is introduced, e.g. rat's hunger: $\mathcal{H}_{\text{combined}} = \mathcal{H}_{\text{position}} \otimes \mathcal{H}_{\text{hunger}}$. This could illustrate features like Heisenberg inequality on macro system (rat).
- Comparing simulations from rat's perspective (MDP, Pacman task) and observer's perspective (current setup).

References

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