Quantum Rat Vol.2: Out-of-the-Box Thinking in a Boxed Environment

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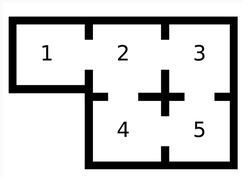
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Introduction

Quantum mechanics (QM) provides a formal framework for modelling uncertainty and dynamic evolution in physical systems. While its mathematical structure is well established, its application beyond microscopic phenomena remains an area of active discussion [4]. This work presents a toy model illustrating core quantum concepts via a simulated rat in a maze. The rat evolves unitarily, while an observer updates their belief state based on rat's position. Varying collapse assumptions reveal the contrast between objective dynamics and subjective knowledge.

Setup: state space

A closed non-transparent box has several rooms. A rat moves between rooms in discrete times. Her position is represented by a room number. Being a normal rodent, she can occupy only one room at a time. Only some rooms are connected by doors, and she can move through one door per step, requiring multiple steps to reach distant rooms. She starts in Room 1 and is naturally drawn to Room 5, where she is usually fed.



The rat's state is represented by a vector in 5-dimensional Hilbert space $\mathcal{H}$:

$$|\psi_{\text{rat}}(t)\rangle = c_1(t)\,|\text{Room}_1\rangle + c_2(t)\,|\text{Room}_2\rangle + c_3(t)\,|\text{Room}_3\rangle + c_4(t)\,|\text{Room}_4\rangle + c_5(t)\,|\text{Room}_5\rangle\,,$$

with coefficients $c_i(t) \in \mathbf{C}$ satisfying the normalization $\sum_{j=1}^{5} |c_j(t)|^2 = 1$. Numeric representation (basis in $\mathcal{H}$) can be chosen as unit vectors: $|\text{Room}_j\rangle \equiv |e_j\rangle$.

Quantum-driven rat

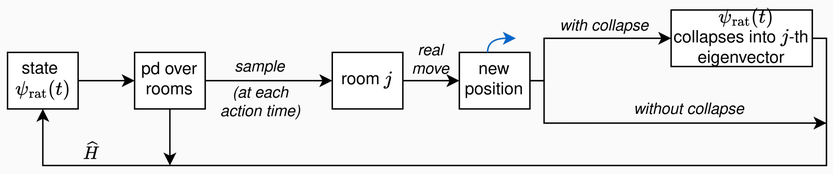
- $|\psi_{\text{rat}}(t)\rangle$ is the state defining the probability distribution of the rat's position in the maze.
- The observable is a Hermitian operator $\hat{\text{A}} : \mathcal{H} \to \mathcal{H}$, whose spectrum describes the possible positions of the rat. According to Born's rule:

$$P\left(\text{rat in the } j\text{-th room} \mid |\psi_{\text{rat}}(t)\rangle\right) = P\left(\alpha_j \mid |\psi_{\text{rat}}(t)\rangle\right) = \left\|\hat{\text{E}}(\alpha_j)\,|\psi_{\text{rat}}(t)\rangle\right\|^2 = |c_j(t)|^2, \tag{1}$$

  where α_j is the j-th eigenvalue, and $\hat{\text{E}}(\alpha_j)$ is the projector onto the direction $|e_j\rangle$.
- At each action time, a random sample is drawn from the probability distribution (1), and the rat **physically moves to a new position**. This new position becomes available to the observer, which constitutes a *measurement*.
- After the movement, the rat may:
  - communicate information about her new position. Then, $|\psi_{\text{rat}}(t)\rangle$ is updated by projecting it onto the eigenvector associated with the new position $|\text{Room}_j\rangle \equiv |e_j\rangle$ (via Lüders formula),
  - do nothing. Then, $|\psi_{\text{rat}}(t)\rangle$ remains unchanged.
- Between two physical movements of the rat, $|\psi_{\text{rat}}(t)\rangle$ evolves unitarily with given Hamiltonian $\hat{\text{H}}$ (initial $|\psi_0\rangle$ is known):

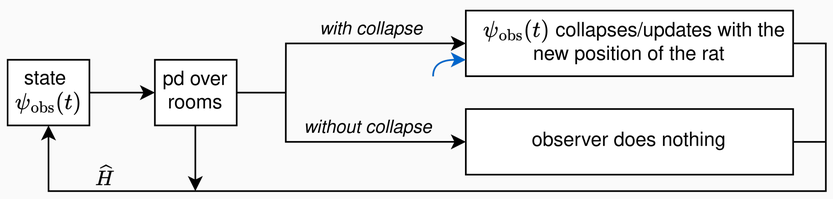
$$|\psi_{\text{rat}}(t_{k+1})\rangle = e^{-i\hat{\text{H}}\Delta t}\,|\psi_{\text{rat}}(t_k)\rangle\,, \quad \Delta t = t_{k+1} - t_k \tag{2}$$

A measurement is made every N evolution steps.



🤔 How to choose $\hat{\text{H}}$ in (2)? Can its elements be interpreted as rewards for the rat?
🤔 How should N and Δt be chosen?

Observer outside of the box

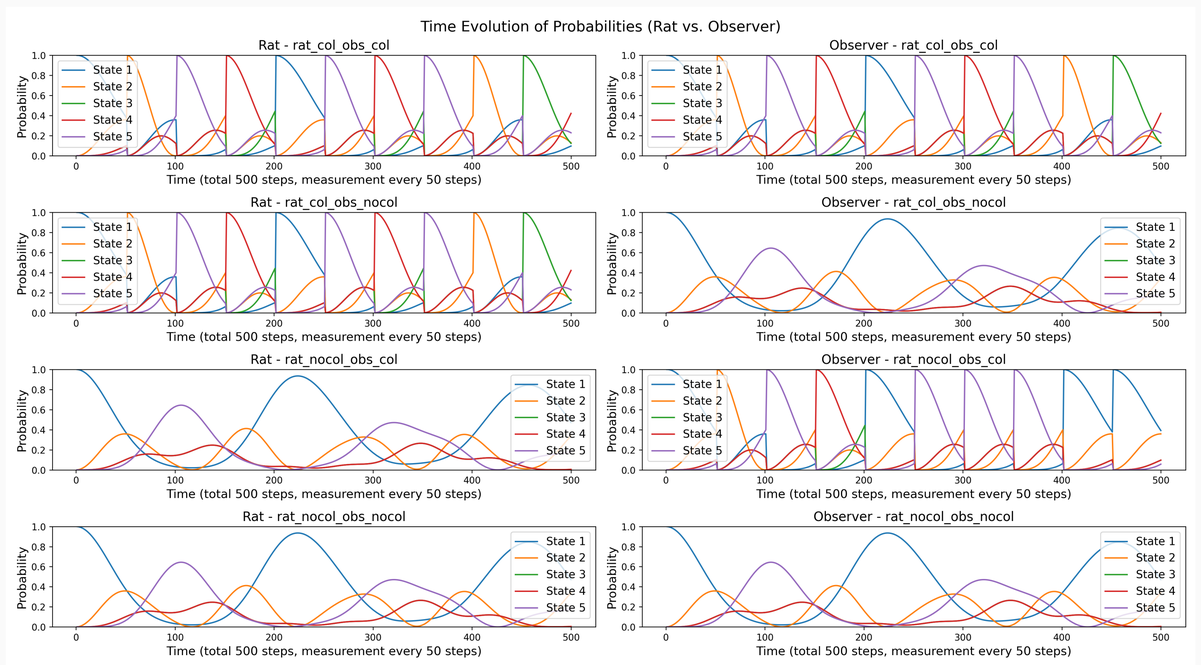


- The observer models the rat's position. His state $|\psi_{\text{obs}}(t)\rangle$ expresses his belief regarding the rat's movement. Knowing $\hat{\text{H}}$ and $|\psi_0\rangle$, he computes probabilities of the rat position based on $|\psi_{\text{obs}}(t)\rangle$ (via (1)).
- The observer
  - *either* updates $|\psi_{\text{obs}}(t)\rangle$ with the measured real position (via Lüders formula),
  - *or* does nothing.
- The observer evolves his $|\psi_{\text{obs}}(t)\rangle$ in time with unitary evolution (like (2)).

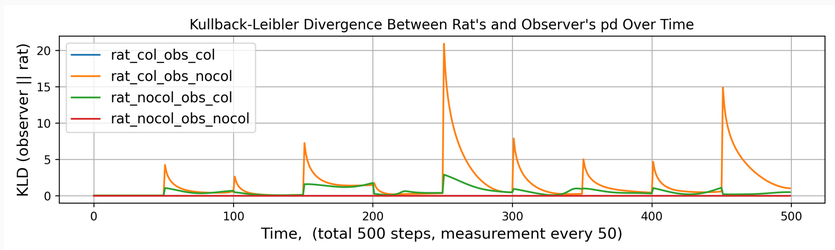
Simulation

- The setup consists of 2 agents:
  Rat moves in the maze accordingly to the probabilities (which are driven by $\hat{\text{H}}$).
  Observer tries to identify probabilities of rat's position ($\hat{\text{H}}$ is his prior knowledge).
- Four scenarios are considered:
  - **Both do collapse**: Both rat and observer update their states based on measurement.
  - **Only the rat does collapse**: the rat updates her state with new position, while observer keeps evolving his state without collapse.
  - **Only observer does collapse**: rat evolves unitarily, while observer updates his state (belief) based on where the rat is.
  - **No collapse**: Both evolve their states unitarily.
- Measurement (in QM sense) applies only to the rat. The observer's "collapse" is epistemic: he is updating his belief about probabilities of the rat's position with information (data) observed by the rat.

Figure shows how probability evolves in time - for each agent in every scenario.



KL divergence compares pds above. In both scenarios, when observer does what the rat does, there is no difference (since $\hat{\text{H}}$, $|\psi_0\rangle$ are the same). Scenario when observer updates his belief, while rat does no collapse, is more "precise".



🤔 What does state function describing {rat + observer} jointly look like?

Results

- This work illustrates the distinction between *objective* probabilities (driving the rat) and *subjective* probabilities (the observer's belief about the rat's state).
- Although the setup assumes an idealized rat whose behavior is governed by a fixed Hamiltonian $\hat{\text{H}}$, it reveals how structurally identical quantum models can lead to different dynamics when viewed from the *inside* (rat) vs. *outside* (observer).
- The model allows exploration of cases where the observer has an incorrect or uncertain initial state $|\psi_0\rangle$, affecting belief updates over time.
- The setup also demonstrates the *Quantum Zeno effect* – frequent observations can cause the rat to "freeze" in a single room.
- The model echoes modern interpretations (e.g., Relational QM, QBism [1]), in which quantum states represent an agent's information or belief about a system — not an objective property of reality. The observer's state reflects such an epistemic stance.

Summary and future of the rat

Even this toy model highlights deep questions about the interpretation of quantum theory – particularly the distinction between the *observer's subjective model* and the *system's underlying dynamics*. The setup allows for future extensions, such as algorithms to estimate the Hamiltonian $\hat{\text{H}}$ governing the system, based on prior knowledge of the task and data collected from observations (work in progress). More broadly, the framework can be adapted to cognitive science experiments. Substituting the simulated "quantum-driven rat" with a real living organism may help test whether certain features of decision-making [3] and preference formation can be described by quantum-like models – including effects of internal intention and external feedback.

References

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